

PROGRAM EVALUATION AND REVIEW TECHNIQUE

BASIC CONCEPTS AND FORMULA

Basic Concepts

1. Program Evaluation and Review Technique

PERT is more relevant for handling such projects which have a great deal of uncertainty associated with the activity durations.

2. Types of Times Estimates

To take these uncertainty into account, three kinds of times estimates are generally obtained. These are:

2.1 The Optimistic Time Estimate

This is the estimate of the shortest possible time in which an activity can be completed under ideal conditions. For this estimate, no provisions for delays or setbacks are made. We shall denote this estimate by t_o .

2.2 The Pessimistic Time Estimate

This is the maximum possible time which an activity could take to accomplish the job. If everything went wrong and abnormal situations prevailed, this would be the time estimate. It is denoted by t_p .

2.3 The Most Likely Time Estimate

This is a time estimate of an activity which lies between the optimistic and the pessimistic time estimates.

The variance is $S_{t2} = \left(\frac{t_p - t_o}{6} \right)^2$

3. Expected time

The expected time (t_e) is the average time taken for the completion of the job. By using beta-distribution, the expected time can be obtained by following formula.

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

4. Probability estimate in PERT

The probability of completing the project by scheduled time is assessed with normal variate Z given by

$$Z = \frac{T_1 - T_{cp}}{S.D.}$$

Where T_1 denotes the duration in which we wish to complete the project and T_{cp} represents the duration on the critical path, S.D. stands for standard deviation of the earliest finish of a network.

5. Project Crashing

It means reduction in project duration. Reduction in duration involves application of additional resources which involves additional cost and at the same time reduction in indirect cost per day. We identify the activities which can be crashed and compare the activity cost slope with indirect cost per day so as to arrive at project duration at optimum cost.

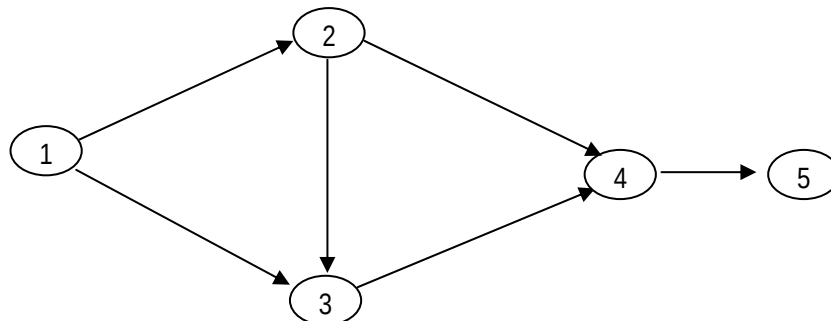
Activity cost slope = (Crash cost – Normal Cost) / (Normal time – Crash time)

6. Resource smoothing

It is used for smoothening the peak resource requirement during different periods of project duration. It is a time scaled diagram of various activities and their float along with resource requirement. Float gives the option of balancing the resources over longer period so that resource requirement is smoothened without much affecting the project duration.

Question 1

The normal time, crash time and crashing cost per day are given for the following network:



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Activity	Normal time (days)	Crash time (days)	Crashing cost (Rs./day)
1-2	18	14	40
1-3	23	22	20
2-3	8	5	60
2-4	10	6	40
3-4	3	2	80
4-5	8	6	50

- (i) Crash the project duration in steps and arrive at the minimum duration. What will be the critical path and the cost of crashing?
- (ii) If there is an indirect cost of Rs. 70 per day, what will be the optimal project duration and the cost of crashing?

Answer

- (i) Critical path

1 - 2 - 3 - 4 - 5 37 days

Paths:

	Normal	Crash
1 - 2 - 4 - 5	36	26
1 - 2 - 3 - 4 - 5	37	27
1 - 3 - 4 - 5	34	30

	Crash Activity	Days,	Cost
Step I	1 - 2	1	40
II	1 - 2	1	40
III	4 - 5	1	50
IV	4 - 5	1	50
V	2 - 3	1	60
VI	1 - 2 & 1 - 3	<u>1</u>	<u>60</u> (40 + 20)
		<u>6</u>	300

Revised critical paths :

$$\left. \begin{array}{l} 1-2-4-5 \\ 1-2-3-4-5 \\ 1-3-4-5 \end{array} \right\} 31 \text{ days}$$

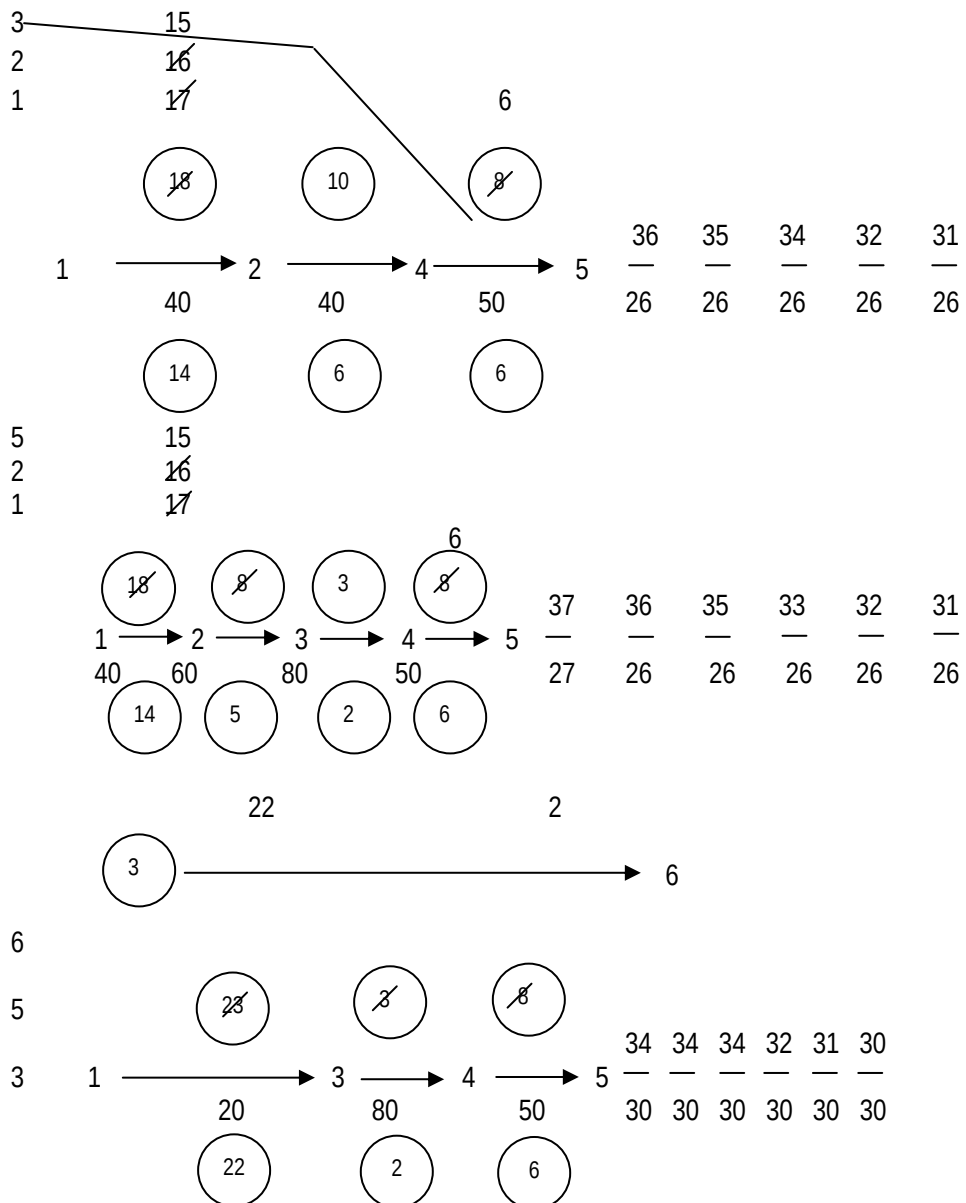
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VII	1 – 2 & 3 – 4	(40 + 80)	$\frac{120}{420}$	30 days
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But VII is not done if indirect cost = 70, which is < 120.

(ii) Project duration is 31 days and cost of crashing is 300.

Workings of Crashing:



Critical Path : 1 – 2 – 3 – 4 – 5 (37 days) Crashing cost

(1)	1 – 2	1 day	40
(2)	1 – 2	1 day	40
(3)	4 – 5	2 days	100
(4)	2 – 3	1 day	60
(5)	1 – 2 & 1 – 3	1 day (40 + 20)	<u>60</u>
			<u>300</u>

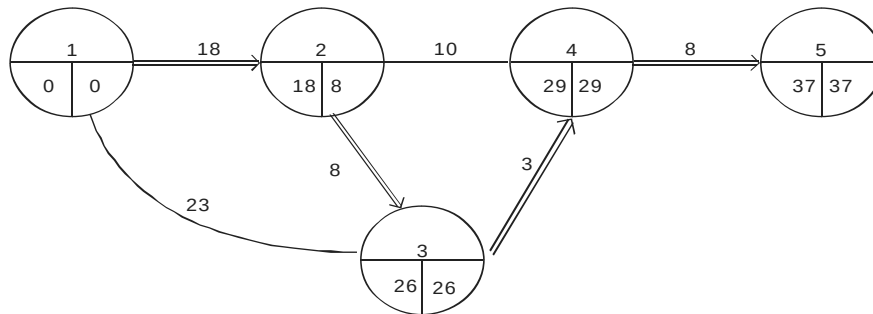
Revised critical Path: 1 – 2 – 4 – 5
 1 – 2 – 3 – 4 – 5
 1 – 3 – 4 – 5

Duration 31 days

(Note: After each crashing a networking diagram has to be drawn and critical path has to be decided).

Alternative Solution:

(i) Network Diagram:



Project duration = 37 days.

Critical Paths :

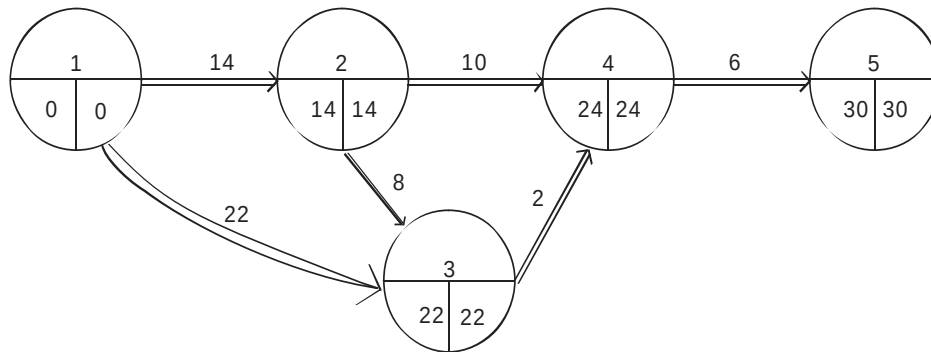
- (i) 1 → 3 → 4 → 5
- (ii) 1 – 2 – 3 – 4 – 5

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Crashing by Steps

Step : 1	Crash activity	Crashing No. of days	Crash cost per day (Rs.)	Crashing cost (Rs.)
	1–2	3	40	120
Step : 2	1–2	1	40	40
	1–3	1	20	20
Step : 3	4–5	2	50	100
Step : 4	3–4	1	80	<u>80</u>
				<u>360</u>

Revised Network



Effective crashing days = 7

Critical Paths:

- (i) 1 – 2 – 4 – 5
- (ii) 1 – 3 – 4 – 5
- (iii) 1 – 2 – 3 – 4 – 5

Project duration = 30 days

Crashing cost = Rs. 360

- (ii) For optimal project duration, we have to consider indirect cost per day i.e., Rs. 70. The crashing cost of activity 3–4 is Rs. 80 which is higher than indirect cost per day. Hence, we may opt it out (Step 4).

In that case, project duration = 31 days.

Crashing cost = Rs. 280.

Saving in indirect cost = $6 \times \text{Rs. } 70 = \text{Rs. } 420$.

Question 2

A project with normal duration and cost along with crash duration and cost for each activity is given below:

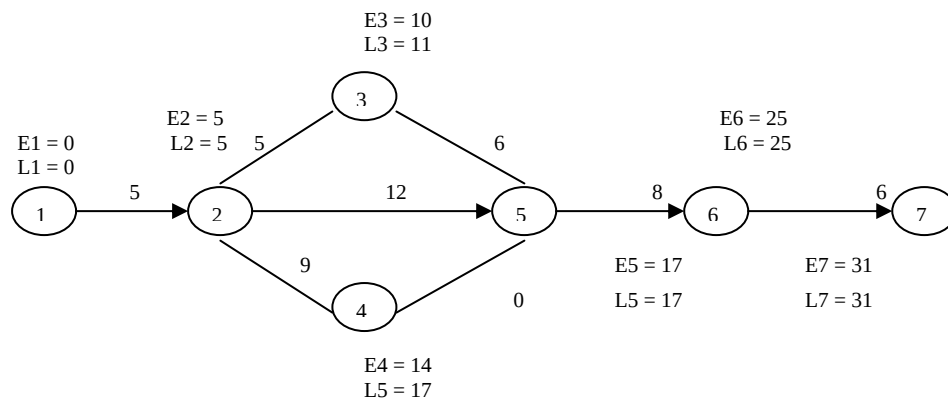
Activity	Normal Time (Hrs.)	Normal Cost (Rs.)	Crash Time (Hrs.)	Crash Cost (Rs.)
1-2	5	200	4	300
2-3	5	30	5	30
2-4	9	320	7	480
2-5	12	620	10	710
3-5	6	150	5	200
4-5	0	0	0	0
5-6	8	220	6	310
6-7	6	300	5	370

Required:

- Draw network diagram and identify the critical path.
- Find out the total float associated with each activity.
- Crash the relevant activities systematically and determine the optimum project completion time and corresponding cost.

Answer

(i) Net work diagram



Path are

- 1-2-5-6-7 = 31 hours, this is critical path
- 1-2-3-5-6-7 = 30 hours
- 1-2-4-5-6-7 = 28 hours

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(ii) Total floats

Activity	Duration hours	Early start	Latest start	Early finish	Latest finish	Total float
1-2	5	0	0	5	5	0
2-3	5	5	6	10	11	1
2-4	9	5	8	14	17	3
2-5	12	5	5	17	17	0
3-5	6	10	11	16	17	1
4-5	0	14	17	14	17	3
5-6	8	17	17	25	25	0
6-7	6	25	25	31	31	0

(iii) Calculation of crashing

Activity	Nt	Nc	Ct	Cc	Slop = $(C_c - N_c) / (N_t - C_t)$
1-2	5	200	4	300	100
2-3	5	30	5	30	0
2-4	9	320	7	480	80
2-5	12	620	10	710	45
3-5	6	150	5	200	50
4-5	0	0	0	0	0
5-6	8	220	6	310	45
6-7	6	300	5	370	70

The critical path activities are 1-2 2-5 5-6 6-7

Slope 100 45 45 70

Two activities cost slope cost is minimum (2-5 and 5-6) but activity 5-6 is common and critical, it also continuing so reduce by 2 hours, then reduce activity 2-5 by one hour.

	Activity	From-to	Project durations	Cost
I	5-6	8-6 hours	$31-2 = 29$	$1840 + (2 \times 45) + (29 \times 50) = 3380$
II	2-5	12-11	$29-1 = 28$	$1840 + 90 + (1 \times 45) + 28 \times 50 = 3375$

After this reduction now two paths are critical 1-2-3-5-6-7 = 28 and 1-2-5-6-7 = 28

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So	1-2	3-5	6-7
	2-5		
Slope cost	100	50+45=95	70

As cost per hour for every alternative is greater than Rs.50 (overhead cost per hour). Therefore, any reduction in the duration of project will increase the cost of project completion. Therefore, time for projects is 28 weeks, minimum cost is Rs.3375.

Question 3

An Engineering Project has the following activities, whose time estimates are listed below:

Activity (i – j)	Estimated duration (in months)		
	Optimistic	Most likely	Pessimistic
1-2	2	2	14
1-3	2	8	14
1-4	4	4	16
2-5	2	2	2
3-5	4	10	28
4-6	4	10	16
5-6	6	12	30

- (i) Draw the project network and find the critical path.
- (ii) Find the expected duration and variance for each activity. What is the expected project length?
- (iii) Calculate the variance and standard deviation of the project length.
- (iv) What is the probability that the project will be completed at least eight months earlier than expected time?
- (v) If the project due date is 38 months, what is the probability of not meeting the due date? Given:

Z :	0.50	0.67	1.00	1.33	2.00
Prob. :	0.3085	0.2514	0.1587	0.0918	0.0228

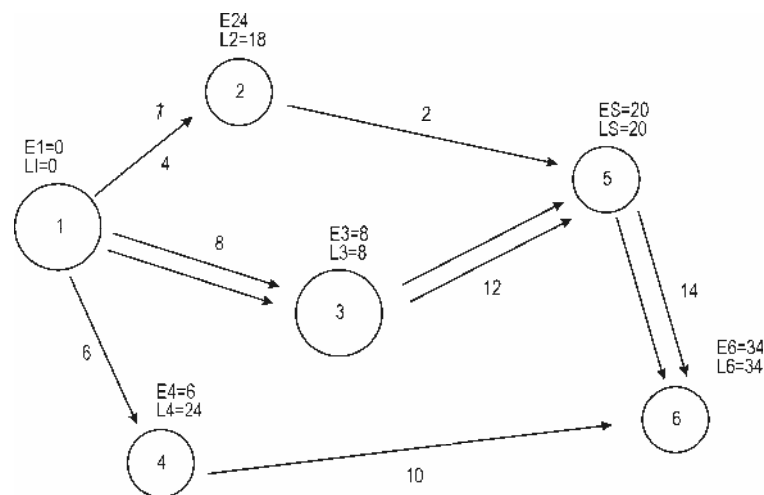
Answer

The earliest and latest expected time for each event is calculated by considering the expected time of each activity as shown in the table below:

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Activity (i - j)	t_0	t_m	t_p	$t_e = (t_0 + 4t_m + t_p) / 6$	$\sigma^2 = \left(\frac{t_p - t_0}{6} \right)^2$
1-2	2	2	14	4	4
1-3	2	8	14	8	4
1-4	4	4	16	6	4
2-5	2	2	2	2	0
3-5	4	10	28	12	16
4-6	4	10	16	10	4
5-6	6	12	30	14	16

(a) The project network is drawn below:

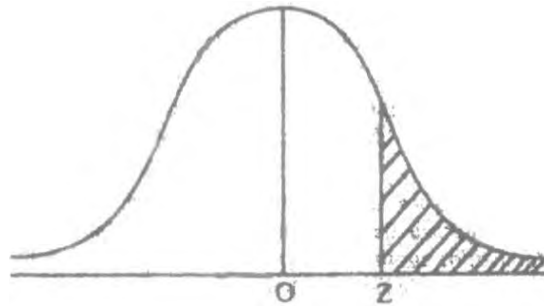


- Critical Path is : 1 – 3- 5 – 6
- The expected duration and variance of each activity is shown in the table above.
The expected project length is the sum of the duration of critical activities.
Hence,
Expected project Length = 8 + 12 + 14 = 34 months
- Variance of the project length is the sum of the variances of critical activities.
Variance of project length = $\sigma^2 = 4 + 16 + 16 = 36$ months
Therefore, Standard Deviation = $\sigma = \sqrt{36} = 6$
- Probability that the project will be completed at least 8 months earlier than the expected time of 34 months is given by

$$\text{Prob.} \left[Z \leq \frac{T_s - T_e}{\sigma_e} = \frac{(34 - 8) - 34}{6} \right] = \text{Prob.}[Z \leq -1.33]$$

But $Z = -1.33$ from the normal distribution table is 0.0918.

Students may please note that the values for the Prob. For a Z value correspond to the shaded area as shown in the diagram below:



Thus, the probability of completing the project within 26 months is 9.18%.

- (v) If the project due date is 38 months, then the probability of not meeting the due date is given by

$$\text{Prob.} \left[Z > \frac{T_s - T_e}{\sigma_e} = \frac{(38 - 34)}{6} \right] = \text{Prob.}[Z > 0.67]$$

But $Z = 0.67$ from the normal distribution is 0.2514.

Thus, the probability of not meeting the due date is 25.14%.

Question 4

A small project consists of jobs as given in the table below. Each job is listed with its normal time and a minimum or crash time (in days). The cost (in Rs. per day) of each job is also given:

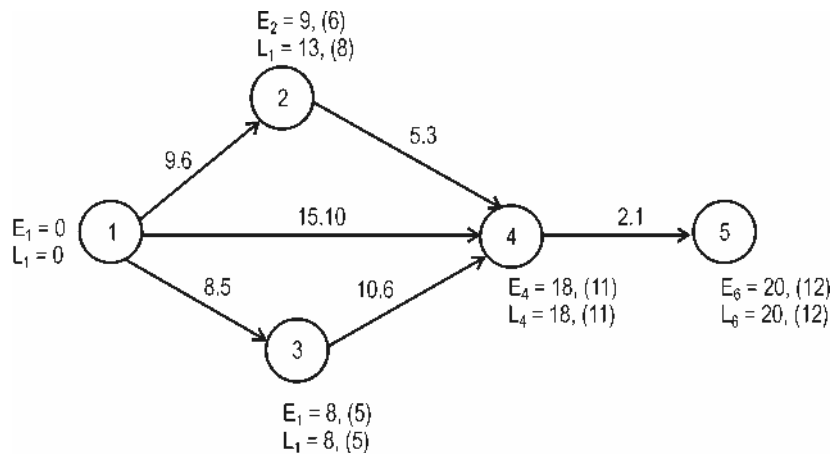
Job (i – j)	Normal duration (in days)	Minimum (crash) Duration (in days)	Cost of Crashing (Rs. per day)
1-2	9	6	20
1-3	8	5	25
1-4	15	10	30
2-4	5	3	10
3-4	10	6	15
4-5	2	1	40

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- (i) What is the normal project length and the minimum project length?
- (ii) Determine the minimum crashing cost of schedules ranging from normal length down to, and including the minimum length schedule. That is, if L = Length of the schedule, find the costs of schedules which are L , $L - 1$, $L - 2$ and so on.
- (iii) Overhead costs total Rs.60 per day. What is the optimum length schedule in terms of both crashing and overhead cost? List the schedule duration of each job for your solution.

Answer

- (i) The required network is given below:



The various paths in the network are:

1 – 2 – 4 – 5 with project duration = 16 days

1 – 4 – 5 with project duration = 17 days

1 – 3 – 4 – 5 with project duration = 20 days

The critical path is 1 → 3 → 4 → 5. The normal length of the project is 20 days and minimum project length is 12 days.

- (ii) Since the present schedule consumes more time than the minimum project length, the duration can be reduced by crashing some of the activities. Also, since the project duration is controlled by the activities lying on the critical path, the duration of some of the activities lying on critical path can be reduced. It is given that overhead cost is Rs.60 per day.

Step I: First, the crashing cost of activity (3, 4) being minimum, the duration of this activity can be compressed from 10 days to 9 days. The total cost for 19 day's schedule = Rs.15 + Rs.19 × 60 = Rs.1,155

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Step II: Since the critical path remains unchanged, the duration of activity (3, 4) can be further reduced from 9 days to 8 days resulting in an additional cost of Rs.15 so that total cost for 18 days schedule = Rs.30 + Rs.60 × 18 = Rs.30 + Rs.1,080 = Rs.1,110.

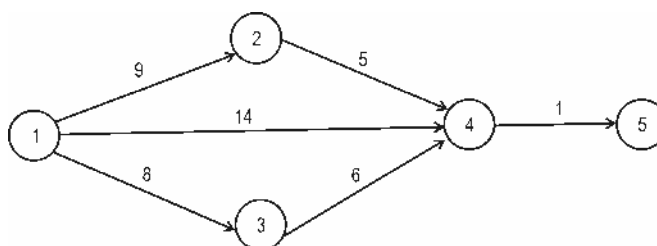
Step III: Continue this procedure till the minimum project length schedule. The calculations are given below:

Normal Project length (days)	Job crashed	Crashing Cost (Rs.)	Overhead cost @ Rs.60 / day	Total Cost. (Rs.)
20	--	--	20×60	1,200
19	3-4	1 × 15 = 15	19×60	1,155
18	3-4	2 × 15 = 30	18×60	1,110
17	3-4	3 × 15 = 45	17×60	1,065
16	4-5	3×15+1×40 = 85	16×60	1,045
15	3-4, 1-4	4×15+1×40+1×30= 130	15×60	1,030
14	1-3, 1-4, 2-4	130+1×30+1×25+1×10=195	15×60	1,035
13	1-3, 1-4, 2-4	195+1×25+1×30+1×10=260	13×60	1,040
12	1-3, 1-4, 1-2	260+25+30+20=335	12×60	1,055

- (iii) Since the total cost starts increasing from 14 days duration onwards, the minimum total cost of Rs.1,030 for the optimum project duration of 15 days occurs for optimum duration of each job as given below:

Job:	(1,2)	(1,3)	(1,4)	(2,4)	(3,4)	(4,5)
Optimum:	9	8	14	5	6	1

Duration (day)



Path 1 → 2 → 4 → 5 = 9 + 5 + 1 = 15 days

Path 1 → 4 → 5 = 14 + 1 = 15 days

Path 1 → 3 → 4 → 5 = 8 + 6 + 1 = 15 days.

Hence, the optimum duration of the project is 15 days.

Question 5

Write short notes on Distinction between PERT and CPM.

Answer

Distinction between PERT and CPM: The PERT and CPM models are similar in terms of their basic structure, rationale and mode of analysis. However, there are certain distinctions between PERT and CPM networks which are enumerated below:

- (1) CPM is activity oriented i.e. CPM network is built on the basis of activities. Also results of various calculations are considered in terms of activities of the project. On the other hand, PERT is even oriented.
- (2) CPM is a deterministic model i.e. it does not take into account the uncertainties involved in the estimation of time for execution of a job or an activity. It completely ignores the probabilistic element of the problem. PERT, however, is a probabilistic model. It uses three estimates of the activity time; optimistic, pessimistic and most likely, with a view to take into account time uncertainty. Thus, the expected duration for each activity is probabilistic and expected duration indicates that there is fifty per probability of getting the job done within that time.
- (3) CPM laces dual emphasis on time and cost and evaluates the trade-off between project cost and project item. By deploying additional resources, it allows the critical path project manager to manipulate project duration within certain limits so that project duration can be shortened at an optimal cost. On the other hand, PERT is primarily concerned with time. It helps the manger to schedule and coordinate various activities so that the project can be completed on scheduled time.
- (4) CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects. PERT is generally used for those projects where time required to complete various activities are not known as prior. Thus, PERT is widely used for planning and scheduling research and development project.

Question 6

A small project is composed of seven activities, whose time estimates are listed below. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity (i-j)	Estimated durations (in days)		
	Optimistic	Most likely	Pessimistic
1-2	2	2	14
1-3	2	8	14
1-4	4	4	16

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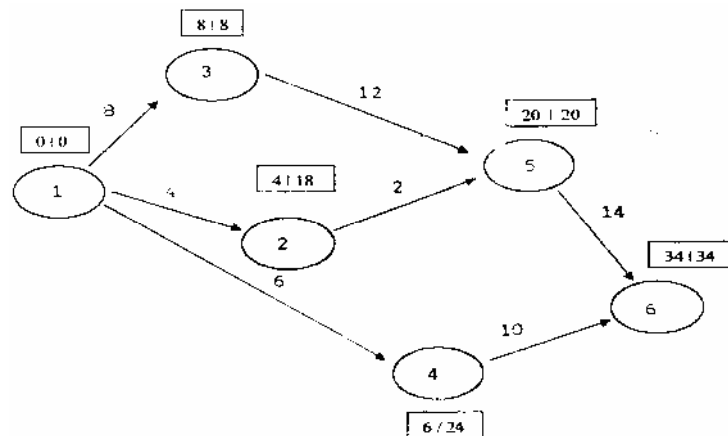
2-5	2	2	2
3-5	4	10	28
4-6	4	10	16
5-6	6	12	30

- (a) Draw the project network.
- (b) Find the expected duration and variance for each activity. What is the expected project length?
- (c) If the project due date is 38 days, what is the probability of meeting the due date ?

Given:	z	0.50	0.67	1.00	1.33	2.00
	P	0.3085	0.2514	0.1587	0.0918	0.0228

Answer

Activity	Estimated durations (in days) = $\frac{a + 4m + b}{6}$			$\sigma^2 = \left(\frac{b-a}{6}\right)^2$	
(i - j)	a	m	b		
1-2	2	2	14	4	4
1-3	2	8	14	8	4
1-4	4	4	16	6	4
2-5	2	2	2	2	0
3-5	4	10	28	12	16
4-6	4	10	16	10	4
5-6	6	12	30	14	16



The critical path is 1-----3-----5-----6

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(b) The expected duration of the project $8+12+14 = 34$ days

(c) Variance of project length is $\sigma^2 = 4 + 16 + 16 = 36$

The standard normal deviate is:

$$Z = \frac{\text{due date} - \text{expected date of completion}}{\sqrt{\text{variance}}}$$

$$Z = \frac{26 - 34}{6} = \frac{8}{6} = -1.33 \text{ probability of meeting the due date is } 0.0918 \text{ or } 9.18\%$$

(d) When due date is 38 days

$$Z = \frac{38 - 34}{6} = \frac{4}{6} = 0.67 \text{ Probability meeting the date is } 0.2514 \text{ or } 25.14\%.$$

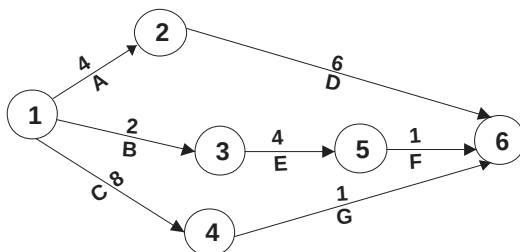
Question 7

The following information is available:

	Activity	No. of days	No. of men required per day
A	1—2	4	2
B	1—3	2	3
C	1—4	8	5
D	2—6	6	3
E	3—5	4	2
F	5—6	1	3
G	4—6	1	8

- (i) Draw the network and find the critical path.
- (ii) What is the peak requirement of Manpower? On which day(s) will this occur?
- (iii) If the maximum labour available on any day is only 10, when can the project be completed?

Answer



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Path	Days
AD	10 CP →
BEF	7
CG	9

Critical Path = 1– 2 – 6

i.e. AD = 10 days.

Peak requirement is 11 men, required on days 7 and 9.

If only 10 men are available on any day, shift F,G to days 10 and 11 and the project can be completed in 11 days.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	A ₂	A ₂	A ₂	A ₂										
					D ₃	D ₃	D ₃	D ₃	D ₃	D ₃				
	B ₃	B ₃	E ₂	E ₂	E ₂	E ₂								
							F ₃							
	C ₅	C ₅	C ₅	C ₅	C ₅	C ₅	C ₅	C ₅						
									G ₈					
	10	10	9	9	10	10	11	8	11	3				
If s/o shift										F ₃	G ₈			
New	10	10	9	9	10	10	8	8	3	6	8			

Question 8

A project consists of seven activities and the time estimates of the activities are furnished as under:

Activity	Optimistic Days	Most likely Days	Pessimistic Days
1–2	4	10	16
1–3	3	6	9
1–4	4	7	16
2–5	5	5	5
3–5	8	11	32
4–6	4	10	16
5–6	2	5	8

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Required:

- (i) Draw the network diagram.
- (ii) Identify the critical path and its duration.
- (iii) What is the probability that the project will be completed in 5 days earlier than the critical path duration?
- (iv) What project duration will provide 95% confidence level of completion ($Z_{0.95} = 1.65$)?

Given

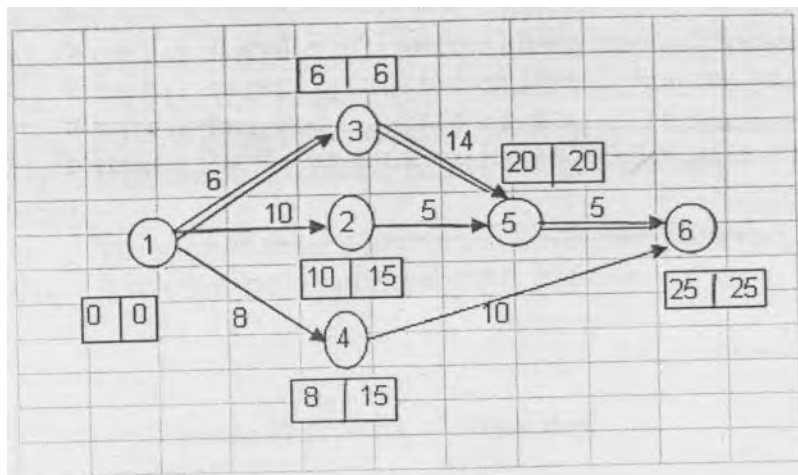
Z	1.00	1.09	1.18	1.25	1.33
Probability	0.1587	0.1379	0.1190	0.1056	0.0918

Answer

Calculation of expected time and variance of each activity:

Activity	Optimistic Days	Most likely Days	Pessimistic Days	Expected Duration	Variance
1-2	4	10	16	10	4
1-3	3	6	9	6	1
1-4	4	7	16	8	4
2-5	5	5	5	5	0
3-5	8	11	32	14	16
4-6	4	10	16	10	4
5-6	2	5	8	5	1

The network diagram is as under:



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Critical Path:	1–3	3–5	5–6			
Duration (days)	6	14	5	= 25 days		
Standard deviation:	1	+	16	+	1	= 18
						$\sqrt{18} = 4.24$

Probability that the project will be completed five days earlier:

$$Z = \frac{20 - 25}{4.24} = -1.18.$$

According to probability values given in the question probability is 11.9%

To obtain 95% confidence level:

$$1.65 = \frac{X - 25}{4.24}$$

$$X - 25 = 6.996$$

$$X = 32 \text{ days}$$

EXERCISE**Question 1**

A small maintenance project consists of the following twelve jobs whose precedence relations are identified with their node numbers.

<i>Job (i, j)</i>	<i>(1, 2)</i>	<i>(1, 3)</i>	<i>(1, 4)</i>	<i>(2, 3)</i>	<i>(2, 5)</i>	<i>(2, 6)</i>
<i>Duration (in days)</i>	10	4	6	5	12	9
<i>Job (i, j)</i>	<i>(3, 7)</i>	<i>(4, 5)</i>	<i>(5, 6)</i>	<i>(6, 7)</i>	<i>(6, 8)</i>	<i>(7, 8)</i>
<i>Duration (in days)</i>	12	15	6	5	4	7

- (i) *Draw an arrow diagram representing the project.*
- (ii) *Calculate earliest start, earliest finish, latest start and latest finish time for all the jobs.*
- (iii) *Find the critical path and project duration.*
- (iv) *Tabulate total float, free float and independent float.*

Answer

- (iii) The critical path is $1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$ and the project is 40 days.
- (iv) Total float, free float and independent float for various activities are calculated in the above table.

Question 2

A project has the following time schedule:

	<i>Time in Weeks</i>	<i>Activity</i>	<i>Time in Weeks</i>
1-2	2	4-6	3
1-3	2	5-8	1
1-4	1	6-9	5
2-5	4	7-9	4
3-6	8	8-9	3
4-7	5		

Construct PERT network and compute:

- (i) *total float for each activity; and*
- (ii) *critical path and its duration.*

Program Evaluation and Review Technique

Answer

The critical path is given by 1-3-6-9 and the project duration is 15 weeks.

Question 3

The following information is given:

Activity	(1-2)	(2-3)	(2-4)	(3-5)	(4-6)	(5-6)	(5-7)	(6-7)
Pessimistic time (in weeks)	3	9	6	8	8	0	5	8
Most likely time (in weeks)	3	6	4	6	6	0	4	5
Optimistic time (in weeks)	3	3	2	4	4	0	3	2

Draw the Network diagram for the above. Calculate:

- (i) Variance to each activity.
- (ii) Critical path and expected project length.
- (iii) The probability that the project will be completed in 23 weeks.

Given that:

Z value :	1.90	1.91	1.92	1.93	1.94
Probability :	0.9713	0.9719	0.9726	0.9732	0.9738

Answer

- (i) Critical path is given by 1 – 2 – 3 – 5 – 6 – 7 and the expected project length is 20 weeks.
- (ii) Variance of the critical path = $\sigma^2 = 0 + 1 + 4/9 + 0 + 1 = 22/9 = 2.444$
Thus, the probability that the project will be completed in 23 weeks is 97.26%.

Question 9

A project consists of eight activities with the following relevant information:

Activity	Immediate Predecessor	Estimated Duration (Days)		
		Optimistic	Most Likely	Pessimistic
A	—	1	1	7
B	—	1	4	7

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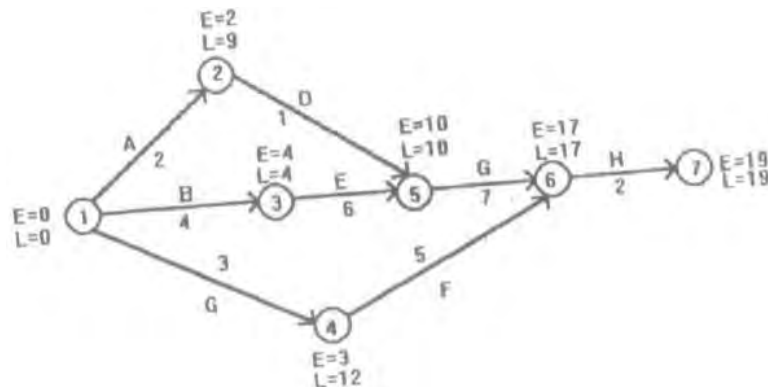
C	—	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D, E	3	6	15
H	F, G	1	2	3

- (i) Draw the PERT network and find out the expected project completion time.
- (ii) What duration will have 95% confidence for project completion?
- (iii) If the average duration for activity F increases to 14 days, what will be its effect on the expected project completion time which will have 95% confidence?

(For standard normal $Z = 1.645$, area under the standard normal curve from 0 to $Z = 0.45$)

Answer

- (i) The required network is drawn below:



The expected time for each activity shown in the network above is calculated in the following table:

Activity	Estimated Duration (Days)			Expected duration $\bar{\tau}$	Variance σ^2
	Optimistic a	Most likely m	Pessimistic b		
A 1-2	1	1	7	2	1
B 1-3	1	4	7	4	1
C 1-4	2	2	8	3	1
D 2-5	1	1	1	1	0
E 3-5	2	5	14	6	4

Program Evaluation and Review Technique

F 4-6	2	5	8	5	1
G 5-6	3	6	15	7	4
H 6-7	1	2	3	2	1/9

The critical path is given by 1 – 3 – 5 – 6 – 7 or B – E – G – H and the expected project completion time is 19 days.

- (ii) The variance for critical path is $1 + 4 + 4 + 1/9 = 82/9$

Standard deviation of critical path = $\sigma_1 = 3.02$ (approx.).

To calculate the project duration which will have 95% chances of its completion, we utilise the given value of Z corresponding to 95% confidence which is 1.645.

Thus, $Z = 1.645$

or $X = 1.645 \times 3.02 + 19 = 23.97$ days = 24 days

Hence, 24 days of project completion time will have 95% probability of its completion.

- (iii) If the average duration for activity F increases to 14 days, then the path 1 – 4 – 6 – 7 i.e. C – F – H will also become critical path with expected project completion time of 19 days. Now, activities C and F are also critical activities. Since we are given only the average duration for activity F, It is assumed that the variance for this activity is zero. Further, since PERT analysis is based on the assumption that the activities are independent in terms of their variance, therefore, standard deviation of critical paths can be computed as:

$$\sigma = \sigma_1$$

We now wish to calculate the expected project completion time that will have 95% confidence level,

$$P(Z < 1.645) = 0.95$$

$$\text{or } X = 19 + 1.645 \times 3.18 = 24.23 \text{ days.}$$

Hence the project duration of 24.23 days will have 95% confidence of completion.

Question 10

A small project consists of seven activities for which the relevant data are given below:

Activity	Preceding activities	Activity Duration (Days)
A	--	4
B	--	7
C	--	6

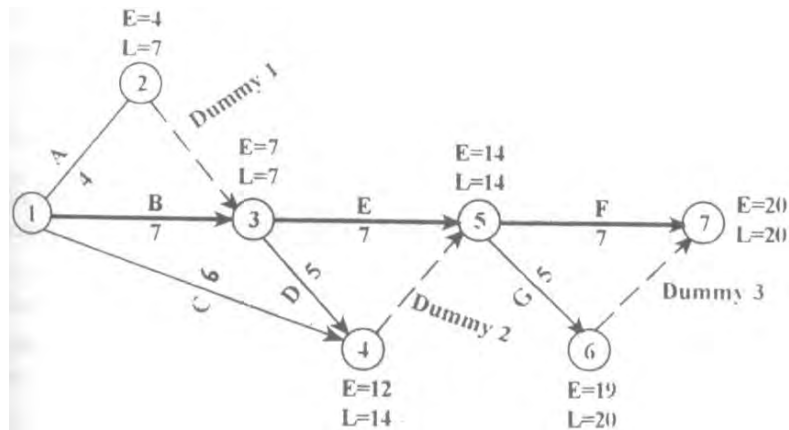
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D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

- Draw the network and find the project completion time.
- Calculate total float for each of the activities.
- Draw the time scaled diagram.

Answer

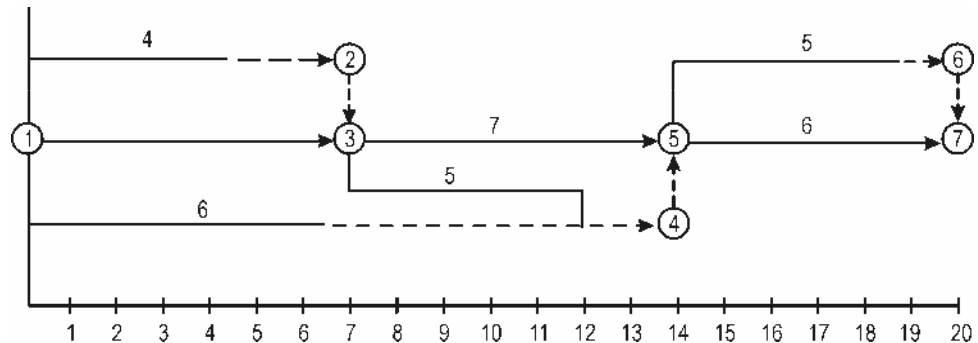
- The required network is given below:



- The total float for various activities is calculated as below:

Activity		Duration	Start		Finish		Total float
			Earliest	Latest	Earliest	Latest	
A	1-2	4	0	3	4	7	3
B	1-3	7	0	0	7	7	0
Dummy 1	2-3	0	4	7	4	7	3
D	3-4	5	7	9	12	14	2
E	3-5	7	7	7	14	14	0
Dummy 2	4-5	0	12	14	12	14	2
F	5-7	6	14	14	20	20	10
G	5-6	5	14	15	19	20	1
Dummy 3	6-7	0	19	20	19	20	1

(iii) The required time scale diagram is drawn below:



Question 13

Define a project and briefly explain the four common implications which characterize a project, and state the five steps of the working methodology of critical path analysis. (May 1997)

Answer

A project can be defined as a set of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within (i) a specified time, (ii) a specified cost and (iii) meeting the performance standards. Examples of a project from fairly diverse fields could be cited. Some of them are given below:

1. Introducing a new product in the market.
2. Construction of a new bridge over a river or construction of a 25 – storied building.
3. Executing a large and complex order on jobbing production.
4. Sending a spacecraft to the mars.

All these projects are characterized by the following set of common implications, although they pertain to widely different fields.

- (i) The Large-scale characteristic: These projects are generally unusually large and complex. Thousands of suppliers, workers and other categories of persons are involved and their efforts have to be coordinated for completion of the project.
- (ii) The non-recurring characteristic: These projects are generally of a one-time nature. Neither in the past, nor in the future they are likely to undertaken substantially in the same form.
- (iii) Uncertain and critical dates: During of the various activities involved in such projects are usually uncertain. Further in such type of projects, many critical dates exists by which operations must be completed in order to complete the entire project on schedule.

- (iv) Completion dead line: The fourth distinct feature of these projects is that there is dead line for the completion of the entire project. In case of any delay in the completion of the project, some penalty is levied for such delay beyond the dead line.

The working methodology of Critical Path Analysis (CPA) which includes both CPM and PERT, consists of following five steps:

1. Analyse and breakdown the project in terms of specific activities and / or events.
2. Determine the interdependence and sequence of specific activities and prepare a network,
3. Assign estimates of time, cost or both to all the activities of the network.
4. Identify the longest or critical path through the network.
5. Monitor, evaluate and control the progress of the project by re-planning, rescheduling and reassignment of resources.